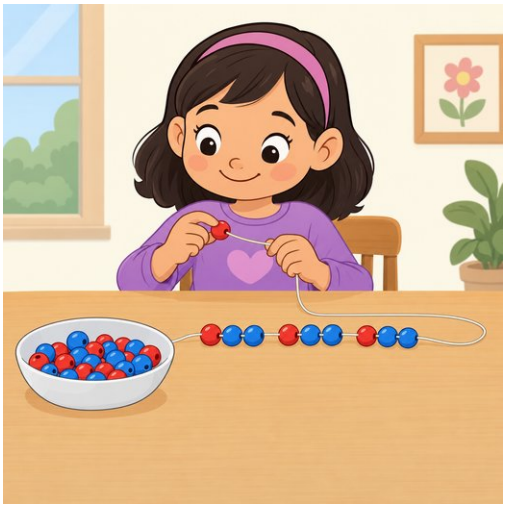
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Grade 1

Problem 1.1 · 3 points — Lia strings beads in a repeating pattern: red, blue, blue, red, blue, blue, red, blue, blue, … How many blue beads are in her first 11 beads?



A) 6
B) 4
C) 7
D) 5
E) 8

Problem 1.2 · 3 points — Tom has 3 balloons. Two of them fly away. Then his mom gives him 4 more. How many balloons does Tom have now?



A) 7
B) 4
C) 3
D) 6
E) 5

Problem 1.3 · 4 points — Two numbers add up to 10. One number is 4 more than the other. What is the bigger number?



A) 7
B) 4
C) 5
D) 6
E) 8

Problem 1.4 · 3 points — Five children stand side by side in a straight row. Mia is exactly in the middle. How many children are to Mia’s left?



A) 4
B) 5
C) 3
D) 2
E) 1

Problem 1.5 · 4 points — A cat is heavier than a rabbit. The rabbit is heavier than a mouse. Who is the lightest?



A) the rabbit
B) the cat
C) the mouse
D) you cannot tell
E) they weigh the same

Problem 1.6 · 5 points — A frog starts on the ground below a 5-step staircase. Each jump takes it up 3 steps, but right after landing it slides down 1 step. The moment it touches step 5, it stops. How many jumps does it need?



A) 3
B) 2
C) 4
D) 1
E) 5

Problem 1.7 · 3 points — Milo moves 1 button from the left box to the right box. Now each box has 3 buttons. How many buttons were in the left box before the move?



A) 4
B) 5
C) 3
D) 2
E) 6

Problem 1.8 · 4 points — Lulu hangs 7 lanterns in a row. Each is lit or dark, the lit-and-dark order is the same from either end, and exactly 3 lanterns are lit. Counting from the left, which lantern must be lit?



A) 5th
B) 3rd
C) 1st
D) 2nd
E) 4th

Problem 1.9 · 4 points — Nina has cards labeled 1, 2, 3, 4, and 5. She uses two different cards to make a two-digit number; how many numbers greater than 30 can she make if the two digits add to 6?



A) 1
B) 3
C) 4
D) 2
E) 5

Problem 1.10 · 5 points — Maja has plenty of 1-cent, 2-cent, and 5-cent coins. In how many different ways can she pay exactly 15 cents using no more than 6 coins? The order of the coins does not matter.



A) 2
B) 4
C) 6
D) 3
E) 5

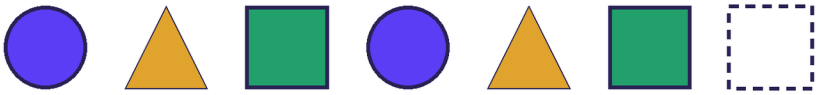
Problem 1.11 · 3 points — Look at these groups: ★ | ●● | ▲▲▲ | ★★★★ | ●●●●● | ?. Which group comes next?



A) ▲▲▲▲▲▲
B) ▲▲▲▲▲▲▲
C) ★★★★★★
D) ▲▲▲▲▲
E) ●●●●●●

Problem 1.12 · 3 points — Look at two shapes sitting side by side. Which pair appears once now, but will appear twice when the empty box is filled to keep the pattern going?

A row of shapes



A) circle then triangle
B) square then circle
C) triangle then square
D) circle then square
E) triangle then circle

Grade 2

Problem 2.1 · 3 points — Mia stamps the pages numbered 17 through 42, but she skips pages 25 through 30 because they already have stamps. How many pages does she stamp?



A) 18
B) 26
C) 20
D) 21
E) 19

Problem 2.2 · 5 points — In a line of children, Tom is 5th from the front and 4th from the back. How many children are in the line?



A) 9
B) 8
C) 10
D) 7
E) 6

Problem 2.3 · 3 points — Mia’s toy building has 3 floors and 12 windows, with the same number on each floor. She adds 2 matching floors, then covers one window on each floor with a sticker; how many windows are left uncovered?



A) 12
B) 18
C) 17
D) 20
E) 15

Problem 2.4 · 4 points — Nia starts at 2 and hops 4 spaces each time, while Bo starts at 4 and hops 6 spaces each time. Which number will they both land on?



A) 22
B) 18
C) 16
D) 28
E) 30

Problem 2.5 · 3 points — Zoe cuts a square sheet of paper into two pieces with ONE straight cut. Which of these can she NOT get?



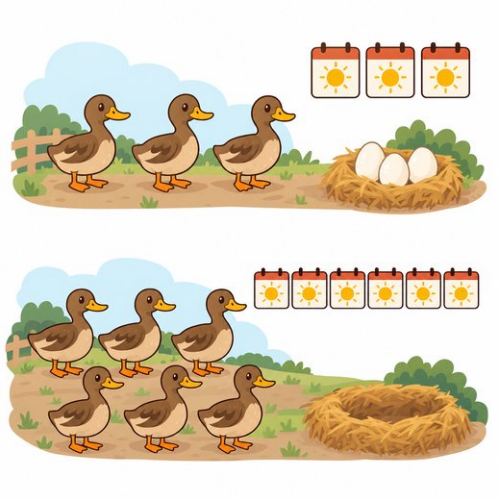
A) a triangle and a pentagon
B) two trapezoids
C) two triangles
D) two squares
E) two rectangles

Problem 2.6 · 4 points — Lila is 4 years older than Sam. Back when Lila was Sam’s age today, Sam was 3. How old is Lila today?



A) 3
B) 7
C) 11
D) 15
E) 8

Problem 2.7 · 5 points — Three ducks lay 3 eggs in 3 days (laying evenly the whole time). How many eggs do 6 ducks lay in 6 days?



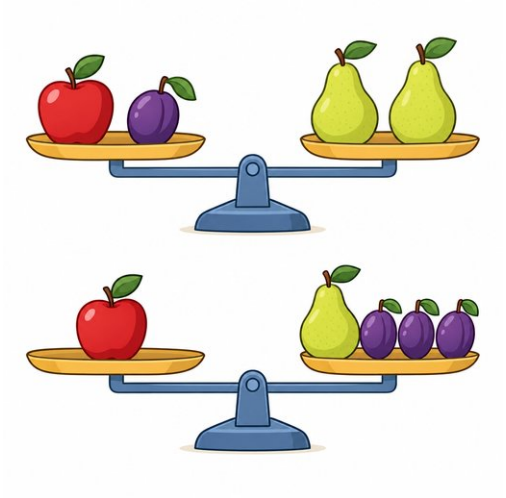
A) 18
B) 12
C) 9
D) 3
E) 6

Problem 2.8 · 3 points — Rosa gave away 3 shells, then found 2 shells. She has 7 shells now. How many did she have at first?



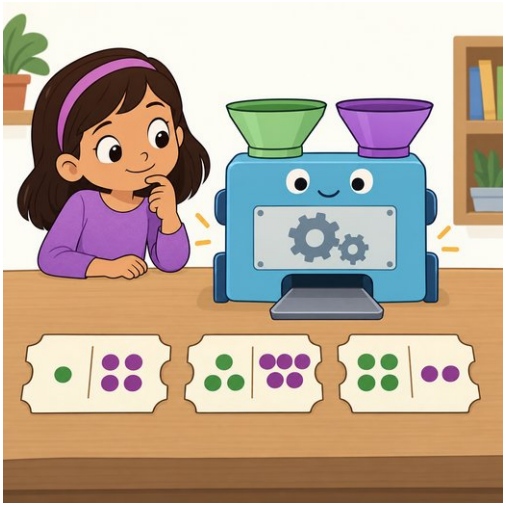
A) 8
B) 9
C) 5
D) 6
E) 12

Problem 2.9 · 4 points — An apple and a plum balance two pears. On another scale, an apple balances a pear and three plums. How many plums balance one pear?



A) 6
B) 3
C) 2
D) 4
E) 5

Problem 2.10 · 3 points — Mia’s number machine makes 1, 4 → 6; 3, 5 → 11; and 4, 2 → 10. Which ticket does not follow its secret rule?



A) 5, 3 → 11
B) 3, 2 → 8
C) 5, 1 → 11
D) 6, 3 → 15
E) 2, 4 → 8

Problem 2.11 · 3 points — Cover two squares so the two you can still see have the same shade, and neither is beside the empty box. Which squares do you cover?

A row of squares



A) the first and third squares
B) the second and fourth squares
C) the first and second squares
D) the second and third squares
E) the third and fourth squares

Problem 2.12 · 4 points — Pip puts 15 pebbles into four pockets. Each pocket has a different number of pebbles, no pocket is empty, and one pocket has exactly 4. What is the most pebbles one pocket can hold?



A) 6
B) 7
C) 9
D) 10
E) 8

Grade 3

Problem 3.1 · 3 points — What is the smallest 3-digit number in which all three digits are different?



A) 100
B) 123
C) 102
D) 120
E) 101

Problem 3.2 · 3 points — A movie starts at 2:40 and lasts 50 minutes. At what time does it end?



A) 4:30
B) 3:40
C) 3:20
D) 3:50
E) 3:30

Problem 3.3 · 3 points — Nina and Leo start with the same even number of stickers, at least 12: Nina gives away 6 and then half of what remains, while Leo gives away half and then 6. How many more stickers does Nina have than Leo?



A) 9
B) 12
C) 0
D) 3
E) 6

Problem 3.4 · 4 points — Two identical squares are placed side by side to make a rectangle. The perimeter of the rectangle is 30. What is the side length of each square?



A) 4
B) 3
C) 5
D) 10
E) 6

Problem 3.5 · 4 points — Mia has a strip of 12 squares numbered 1 to 12 from left to right. She folds the strip exactly in half, then folds the shorter strip exactly in half again; what is the sum of the four numbers in the final stack containing square 2?



A) 28
B) 25
C) 13
D) 24
E) 26

Problem 3.6 · 4 points — I think of a number. I double it, then add 3, and I get 15. What was my number?



A) 9
B) 5
C) 7
D) 6
E) 8

Problem 3.7 · 4 points — How many two-digit numbers have digits that add up to 5?



A) 5
B) 4
C) 6
D) 7
E) 3

Problem 3.8 · 5 points — Five friends share 50 marbles, each getting a different positive number. The biggest share is twice the smallest; what is the largest possible middle share when the five shares are put in order?



A) 12
B) 10
C) 9
D) 11
E) 14

Problem 3.9 · 5 points — Amy, Ben, Cy, and Dee each take a different card numbered 1 through 6, leaving two cards behind; Amy and Ben's cards add to 7, while Cy and Dee's add to 8. Both leftover cards are even, and Amy's card is bigger than Cy's; what number is on Amy's card?



A) 5
B) 6
C) 1
D) 4
E) 3

Problem 3.10 · 3 points — A toy crab starts on shell 5 in a row of shells numbered 1 to 11. It takes exactly four steps, each to a neighboring shell; which shell could it finish on?



A) 7
B) 8
C) 6
D) 4
E) 11

Problem 3.11 · 3 points — A frog has jump cards marked 1, 2, 3, 4, and 5; each card tells how far to jump, and it uses each card once. Starting on 6, it lands on 10, 11, 16, and 18—where will it land after its last jump?



A) 21
B) 24
C) 20
D) 22
E) 19

Problem 3.12 · 3 points — Maya picks three spots in the row. How many different groups of spots can she pick that include both kinds of shapes?

A row of shapes



A) 6
B) 9
C) 10
D) 8
E) 3

Grade 4

Problem 4.1 · 3 points — Lena puts the same whole number in both boxes to make $84 - 5 \times \square = 48 - 2 \times \square$. What number goes in each box?



A) 0
B) 18
C) 12
D) 36
E) 24

Problem 4.2 · 4 points — Mara multiplies two whole numbers and gets 24. When she adds the same two numbers she gets 11. What is the DIFFERENCE of the two numbers?



A) 10
B) 2
C) 5
D) 4
E) 23

Problem 4.3 · 5 points — A three-digit number has digits that add up to 26. How many such numbers are there?



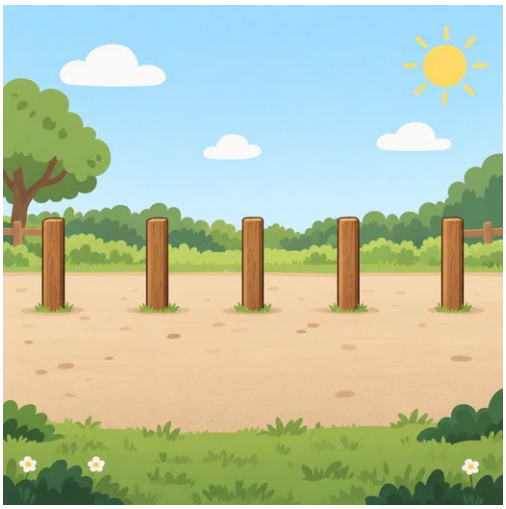
A) 2
B) 3
C) 4
D) 1
E) 5

Problem 4.4 · 3 points — A rope 12 m long is cut into pieces that are each 2 m long. How many cuts are needed?



A) 3
B) 6
C) 4
D) 7
E) 5

Problem 4.5 · 4 points — Five posts stand in a straight row. The distance between one post and the next is 3 m. How far is it from the first post to the last post?



A) 12 m
B) 15 m
C) 9 m
D) 10 m
E) 18 m

Problem 4.6 · 3 points — A toy dragonfly has four wings. Tia puts five stickers on its wings, each on exactly one wing; which statement MUST be true?



A) Some wing has at least two stickers.

B) Some wing has no stickers.

C) Some wing has three stickers.

D) Every wing has a sticker.

E) Exactly one wing has two stickers.

Problem 4.7 · 4 points — Five lamps stand in a row, all off. Bea makes exactly three moves; each move switches two neighboring lamps, turning each on lamp off and each off lamp on. Which row could she make, from left to right?



A) on-on-off-on-on

B) on-on-on-off-off

C) on-off-on-off-off

D) off-on-off-on-off

E) on-off-on-on-on

Problem 4.8 · 3 points — A toy starts on space 1 of a circular board numbered 1 to 8 clockwise. Each dance moves it 5 spaces clockwise, then 2 spaces counterclockwise; where is it after 47 dances?



A) 5
B) 2
C) 4
D) 6
E) 3

Problem 4.9 · 4 points — A 12 cm-wide, 8 cm-tall rectangle has a triangular piece cut from its bottom-left corner. The piece reaches 4 cm along the bottom and 8 cm up the left side. A vertical line divides the remaining shape into two equal areas. How far is the line from the rectangle’s original left side?



A) 4 cm
B) 6 cm
C) 8 cm
D) 7 cm
E) 5 cm

Problem 4.10 · 5 points — Ola writes every number from 1 to 9999. In the numbers whose digits add up to 14, how many times does she write the digit 7?



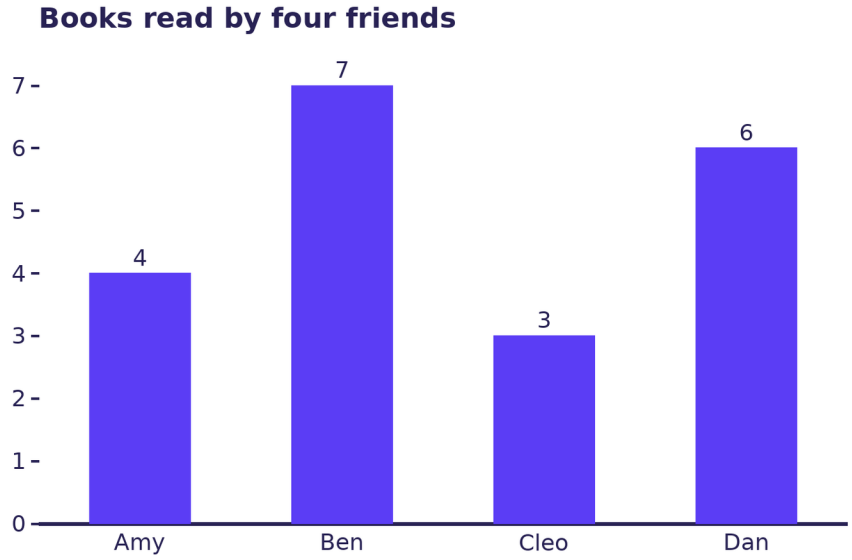
A) 36
B) 144
C) 112
D) 138
E) 140

Problem 4.11 · 3 points — Mia’s number box accepts 9, 25, 49, and 81, but rejects 16, 35, 64, and 99. Which number below would it accept?



A) 121
B) 144
C) 120
D) 143
E) 100

Problem 4.12 · 3 points — If the four friends keep reading until they have all read the same number of books, what is the fewest extra books they must read altogether?



A) 4
B) 8
C) 7
D) 3
E) 1

Grade 5

Problem 5.1 · 3 points — Which of these fractions is the largest?



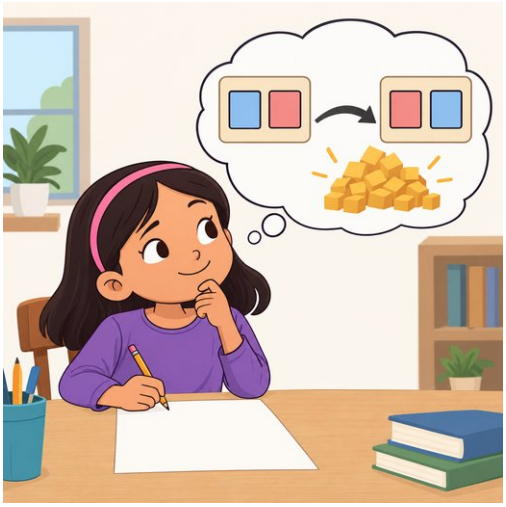
A) 2/5
B) 5/11
C) 1/2
D) 4/9
E) 3/7

Problem 5.2 · 3 points — A positive whole number gives a quotient and a remainder when divided by 7. When the same number is divided by 8, the quotient and remainder switch places. What is the number?



A) 62
B) 54
C) 47
D) 56
E) 55

Problem 5.3 · 4 points — Nina's two-digit number becomes exactly 27 LARGER when its digits are reversed. What is the smallest number Nina could have written?



A) 14
B) 12
C) 13
D) 25
E) 36

Problem 5.4 · 3 points — A rectangular garden has posts 2 m apart around its edge and along a fence joining the midpoints of its long sides, with posts at every corner and at both ends of the middle fence. There are 47 different posts in all, and each long side has 13 posts; how long is a short side?

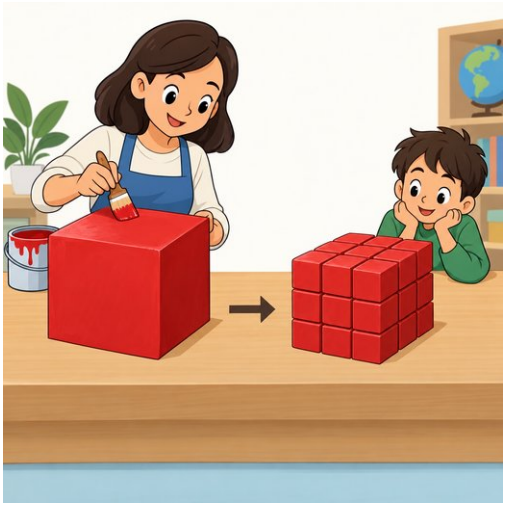
A) 12 m
B) 20 m
C) 14 m
D) 16 m
E) 18 m

Problem 5.5 · 4 points — Eight kids line up: the first five average 11 points, the last five average 15 points, and all eight average 12 points. What is the average score of the middle two kids?



A) 13
B) 34
C) 17
D) 15
E) 12

Problem 5.6 · 4 points — A large cube is painted red on the outside and then cut into 27 equal small cubes (3 × 3 × 3). How many small cubes have exactly ONE red face?



A) 26
B) 8
C) 12
D) 6
E) 1

Problem 5.7 · 5 points — Ada and Bo play three rounds with marbles: first Ada gives Bo as many as Bo already has, then Bo gives Ada as many as Ada now has, and finally Ada gives Bo as many as Bo now has. If they end with 24 marbles each, how many did Ada start with?



A) 18
B) 33
C) 24
D) 15
E) 36

Problem 5.8 · 3 points — Five tiny dragon nests can each hold at most 3 shiny pebbles. A dragon puts 13 pebbles in the nests; what is the smallest possible number of nests holding exactly 3 pebbles?



A) 3
B) 1
C) 4
D) 2
E) 5

Problem 5.9 · 4 points — Eight firefly lanterns are off. Each spell flips exactly three different lanterns (on becomes off and off becomes on); after exactly three spells, what is the greatest number that can be on?



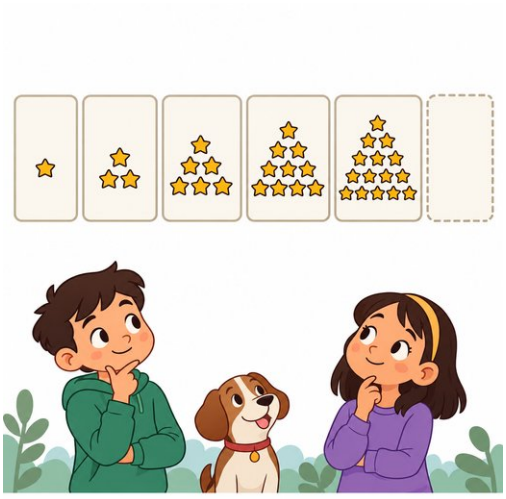
A) 3
B) 5
C) 8
D) 6
E) 7

Problem 5.10 · 5 points — Using 0, 1, 2, 3, and 4 exactly once, make five-digit numbers whose last two digits also form a two-digit number. How many have a larger two-digit number at the start than at the end?



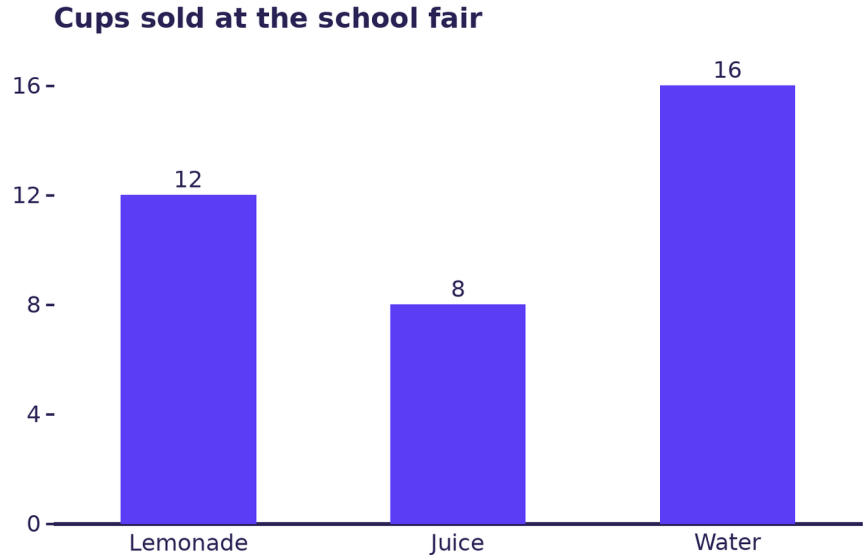
A) 12
B) 36
C) 60
D) 48
E) 72

Problem 5.11 · 4 points — What comes next in the pattern: A, C, F, J, O, ?



A) U
B) W
C) T
D) S
E) V

Problem 5.12 · 4 points — The chart shows cups of lemonade, juice, and water sold at the fair. If all the cups are put onto as many identical trays as possible, with every tray holding all three drinks, how many cups will be on each tray?



A) 36
B) 9
C) 4
D) 12
E) 18

Grade 6

Problem 6.1 · 3 points — A jacket costs $80. Its price is reduced by 25%. What is the new price?



A) $55
B) $20
C) $60
D) $75
E) $65

Problem 6.2 · 3 points — In the morning the temperature was −7 °C. By noon it had risen by 12 degrees. What was the temperature at noon?



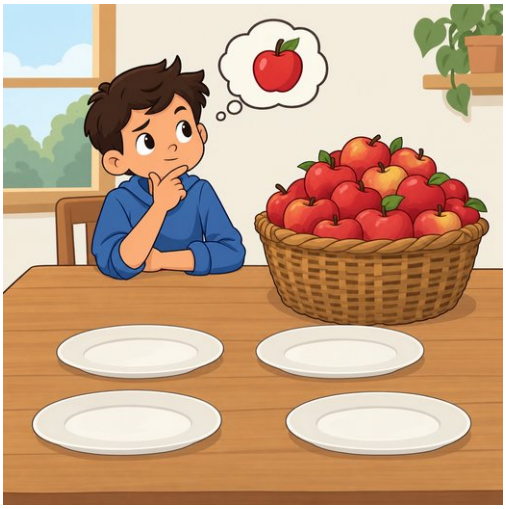
A) 4 °C
B) 19 °C
C) −5 °C
D) −19 °C
E) 5 °C

Problem 6.3 · 3 points — Which fraction is exactly halfway between 1/3 and 1/2?



A) 1/4
B) 4/10
C) 2/5
D) 5/12
E) 3/8

Problem 6.4 · 5 points — How many four-digit perfect squares leave a remainder of 1 when divided by every whole number from 2 through 7?



A) 13
B) 2
C) 3
D) 8
E) 1

Problem 6.5 · 4 points — Lina and Mo have stickers in the ratio 3 : 5. Lina gives Mo 3 stickers, and now Mo has twice as many as Lina. How many stickers did Lina have at first?



A) 30
B) 24
C) 27
D) 72
E) 45

Problem 6.6 · 4 points — The sum of five consecutive whole numbers is 65. What is the LARGEST of the five numbers?



A) 17
B) 13
C) 14
D) 15
E) 16

Problem 6.7 · 4 points — The length of a rectangle is increased by 10% and its width is decreased by 10%. What happens to its area?



A) it shrinks by 1%
B) it grows by 1%
C) it stays the same
D) it shrinks by 10%
E) it grows by 10%

Problem 6.8 · 3 points — Nora’s 3-by-3 tile coaster has exactly five green tiles. Its colors look unchanged after she turns it halfway around (180°); which tile must be green?



A) Center tile
B) Middle-left tile
C) Top-middle tile
D) Top-left tile
E) Bottom-right tile

Problem 6.9 · 5 points — Four islanders are each either a knight (always tells the truth) or a liar (always lies): Ada says, “Exactly one of Ben, Cora, and Drew is a knight”; Ben says, “Ada and Cora are the same kind”; Cora says, “Ben and Drew are different kinds”; and Drew says, “Ada is a liar.” How many of the four are knights?



A) It cannot be determined

B) 2 knights

C) 4 knights

D) 1 knight

E) 3 knights

Problem 6.10 · 4 points — A tinkerer has 5 two-cube rods and 7 single cubes; she may split a rod into two singles or join two singles into a rod. Which total number of separate pieces can she NEVER have?



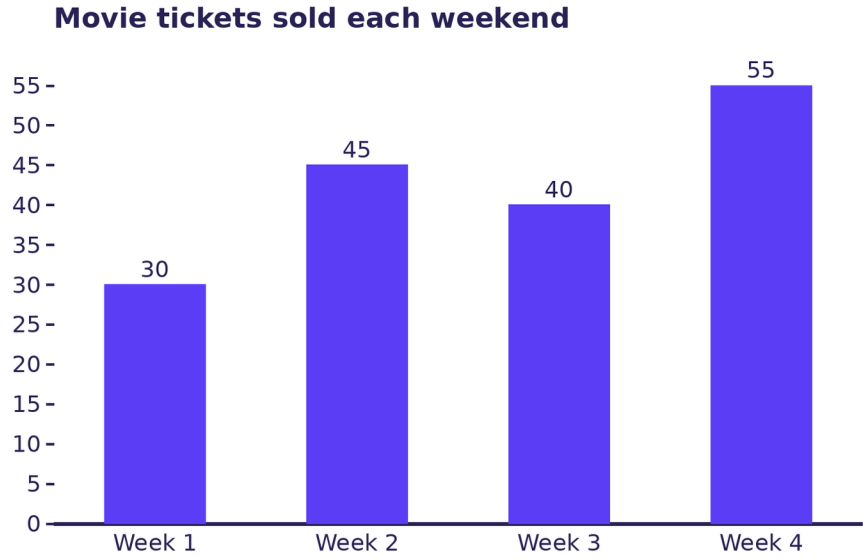
A) 12
B) 11
C) 9
D) 10
E) 8

Problem 6.11 · 4 points — I made an odd four-digit number using 1, 2, 3, and 4 once each. Swapping its thousands and tens digits makes it 1,980 larger; swapping its hundreds and ones digits makes it 198 smaller. What number did I make?



A) 2341
B) 1243
C) 2143
D) 4321
E) 1432

Problem 6.12 · 4 points — If you choose two different weeks from the chart, how many pairs have an average strictly greater than the average for all four weeks?



A) 4
B) 2
C) 1
D) 3
E) 0

Grade 7

Problem 7.1 · 3 points — A machine doubles a two-digit number, adds 7, then swaps the digits of the result. It runs this routine twice and displays 53. What number went in first?

A) 14
B) 32
C) 17
D) 28
E) 23

Problem 7.2 · 3 points — What is the last digit of 2^2026 + 2^2027?



A) 0
B) 8
C) 4
D) 6
E) 2

Problem 7.3 · 4 points — A shirt's price is raised by 20%, and later the new price is reduced by 20%. Compared to the original price, the final price is:



A) 4% higher
B) the same
C) 4% lower
D) 10% lower
E) 2% lower

Problem 7.4 · 5 points — Write down any nine consecutive positive whole numbers, cross out one, and multiply the other eight. Which number is guaranteed to divide the product?



A) 432
B) 144
C) 720
D) 288
E) 1008

Problem 7.5 · 3 points — Seven bowls sit in a row and hold 44 marbles altogether. Each bowl has a positive whole number of marbles, and neighboring bowls hold different amounts. What is the greatest possible number of marbles in the least-filled bowl?



A) 5
B) 7
C) 4
D) 6
E) 3

Problem 7.6 · 3 points — Maya walks once around a triangular park, turning left at each corner to follow its sides. Her first turn is 30° larger than her second, and her third turn is 140°; how large is her second turn?



A) 80°
B) 5°
C) 110°
D) 95°
E) 125°

Problem 7.7 · 4 points — A convex polygon has exactly 20 diagonals. Two vertices joined by a diagonal are colored blue; how many diagonals have no blue endpoint?

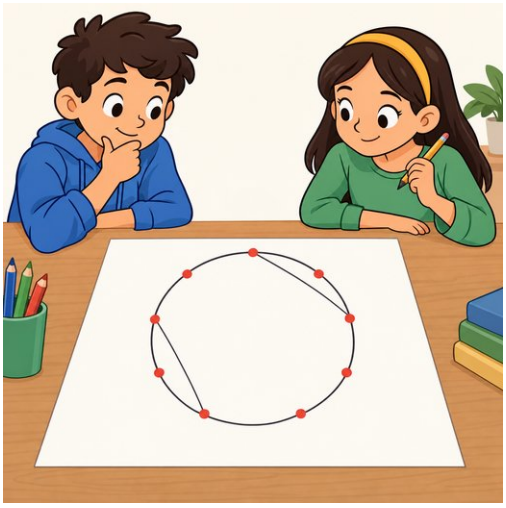
A) 12
B) 10
C) 9
D) 11
E) 15

Problem 7.8 · 4 points — Ala walks to school at 4 km/h and returns home by the same route at 6 km/h. What is her average speed for the whole round trip?



A) 4.8 km/h
B) 5 km/h
C) 4.6 km/h
D) 5.2 km/h
E) 4.4 km/h

Problem 7.9 · 5 points — Nine dots lie on a circle, and every pair is joined by a straight segment. If no three segments meet at the same point inside the circle, how many distinct crossing points are there?

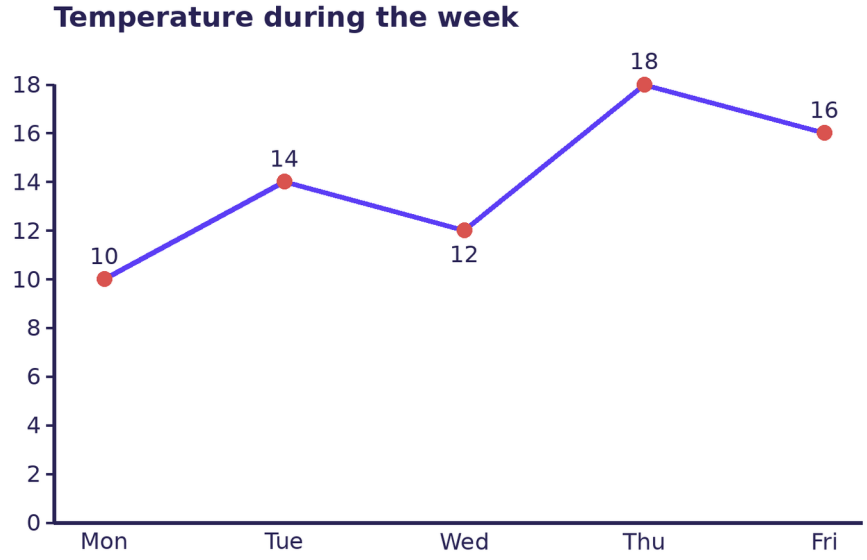


A) 84
B) 126
C) 630
D) 378
E) 252

Problem 7.10 · 4 points — One number in this chain has been smudged: 2, 5, 12, 29, __, 169, 408. What number was it?

A) 70
B) 71
C) 41
D) 58
E) 63

Problem 7.11 · 4 points — Erase one dot from Tuesday, Wednesday, or Thursday, then join its two neighbors with a straight line. Which dot makes the area beneath the graph shrink the most? Give its original temperature.



A) 12 degrees
B) 18 degrees
C) 14 degrees
D) 16 degrees
E) 10 degrees

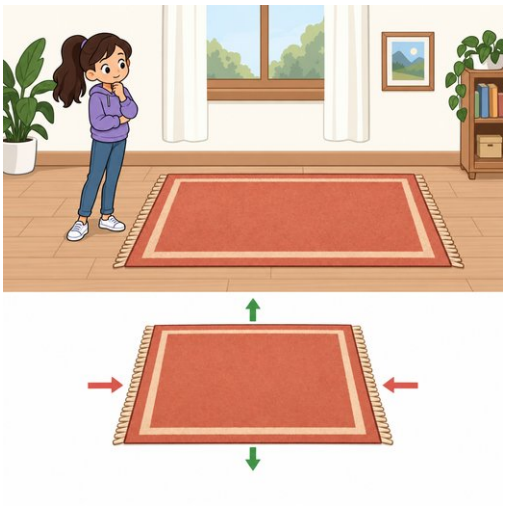
Problem 7.12 · 4 points — Mila draws cards numbered 1 to 12, without looking. What is the fewest cards she must draw to be certain that two of their numbers differ by exactly 3?



A) 9
B) 5
C) 6
D) 8
E) 7

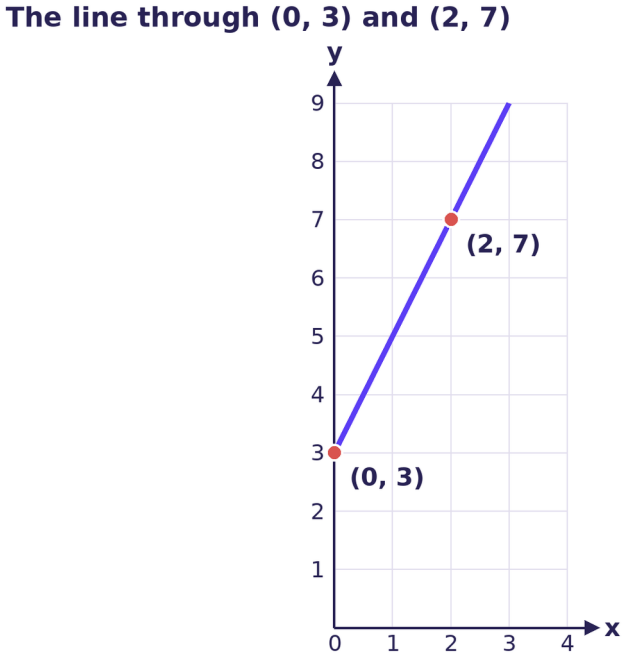
Grade 8

Problem 8.1 · 3 points — A rectangle has a perimeter of 56 cm. Its area increases by 20 cm² if its longer side is shortened by 2 cm and its shorter side is lengthened by 2 cm. What was its original area?



A) 180
B) 192
C) 160
D) 176
E) 196

Problem 8.2 · 4 points — Start at (0, 3) and travel along the drawn line toward (2, 7). When your vertical climb equals the horizontal distance you still have to go, what fraction of the segment between the labeled points have you traveled?



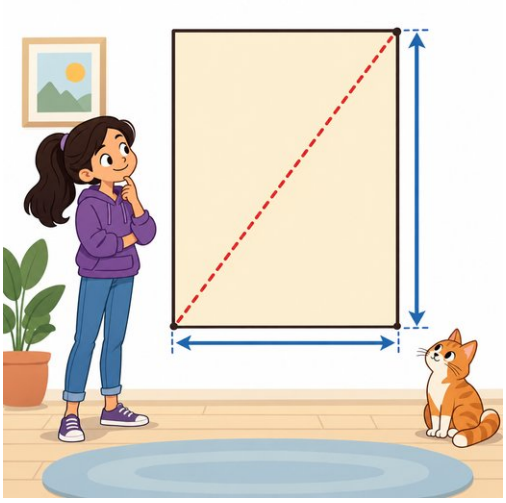
A) 1/2
B) 2/3
C) 1/3
D) 3/4
E) 1/4

Problem 8.3 · 4 points — The equations $x^2 - ax + 24 = 0$ and $x^2 - bx + 36 = 0$ each have two distinct positive integer solutions, and they share one solution. What is the smallest possible value of a + b?



A) 34
B) 22
C) 26
D) 23
E) 29

Problem 8.4 · 3 points — A rectangle is 6 wide and 8 tall. How long is its diagonal?



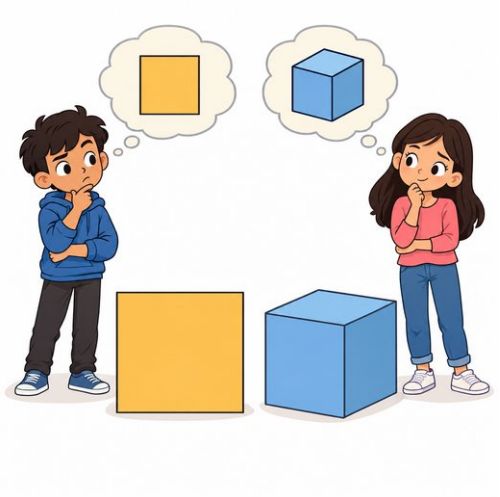
A) 11
B) 14
C) 12
D) 9
E) 10

Problem 8.5 · 5 points — A 7×7 grid of unit squares loses its four corner squares and its center square. What is the greatest number of 1×2 dominoes you can cut from what remains, in either orientation, if you may leave some squares unused?



A) 18
B) 20
C) 19
D) 22
E) 21

Problem 8.6 · 3 points — A positive whole number has both a whole-number square root and a whole-number cube root. If those roots add up to 36, what is the number?



A) 64
B) 4096
C) 1296
D) 729
E) 46656

Problem 8.7 · 5 points — N is the smallest positive integer such that N ÷ 2 is a perfect square and N ÷ 3 is a perfect cube. What is N?



A) 216
B) 648
C) 324
D) 108
E) 432

Problem 8.8 · 4 points — Three different-colored fair six-sided dice are rolled. What is the probability that at least one die shows the average of the other two?



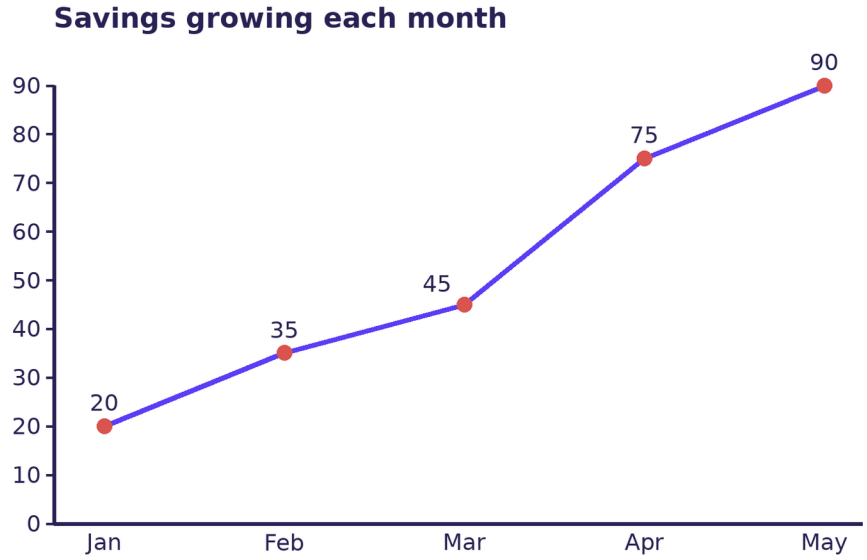
A) 7/36
B) 1/6
C) 1/4
D) 1/12
E) 1/36

Problem 8.9 · 4 points — For every positive integer n, Mira makes two dot cards: an n×n square and a staircase with rows of 1, 2, …, n dots. She sorts all the cards by dot count, keeping tied cards separately; how many dots are on the 100th card?



A) 1764
B) 1681
C) 1770
D) 1849
E) 1711

Problem 8.10 · 4 points — Imagine the four monthly savings increases happened in a different order, while January’s savings stayed the same. Which month’s plotted total could never be made lower by reordering the increases, but could be made higher?



A) April
B) March
C) February
D) May
E) January

Problem 8.11 · 3 points — A chameleon’s scarf has three patches numbered 1, 2, and 3. Each patch can be gold, teal, pink, or purple; how many different scarves have at least one gold patch?



A) 37
B) 36
C) 27
D) 48
E) 64

Problem 8.12 · 4 points — An alien lines up moonstones numbered 1 through 8, using each number once, and writes the sum on every gap between neighbors. If the seven sums add to 69, which two numbers are at the ends?



A) 7 and 8
B) 3 and 4
C) 1 and 3
D) 2 and 3
E) 1 and 2

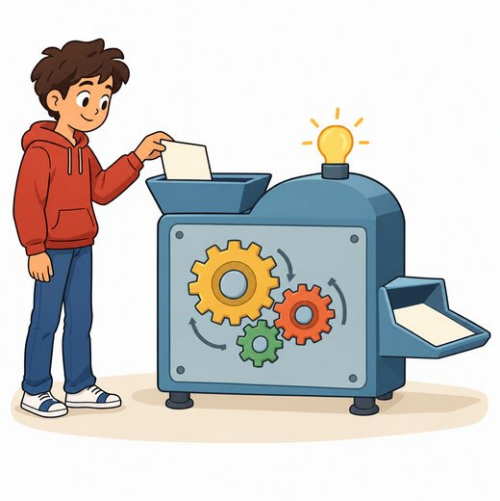
Grade 9

Problem 9.1 · 3 points — What is the sum of all real values of x that make $(x - 4)(x + 2) = (x - 4)(2x - 4)$ true?



A) 6
B) 2
C) 10
D) -2
E) 4

Problem 9.2 · 3 points — A quadratic function gives $f(-2)=10$ and $f(2)=-2$. What is $f(-3)-f(3)$?



A) -12
B) 0
C) -18
D) 18
E) 12

Problem 9.3 · 4 points — For which value of k do $y = kx + 1$, $y = 2x + k - 1$, and $y = x + 3$ form three distinct lines through one point?



A) 1
B) 2
C) 3
D) -1
E) 4

Problem 9.4 · 5 points — Three real numbers x, y, and z satisfy $x+y+z=9$ and $(x-y)^2+(y-z)^2+(z-x)^2=18$. What is $(x+y)(y+z)(z+x)+xyz$?



A) 243
B) 216
C) 162
D) 189
E) 200

Problem 9.5 · 3 points — A train travels at a constant speed. It takes 24 seconds to clear a 280 m tunnel and 32 seconds to clear a 440 m tunnel, each timed from when the train's front enters to when its back exits. How long is the train?



A) 280 m
B) 160 m
C) 480 m
D) 330 m
E) 200 m

Problem 9.6 · 4 points — What is the remainder when $1 + 2 + 2^2 + \cdots + 2^{2026}$ is divided by $2^{1013} + 3$?



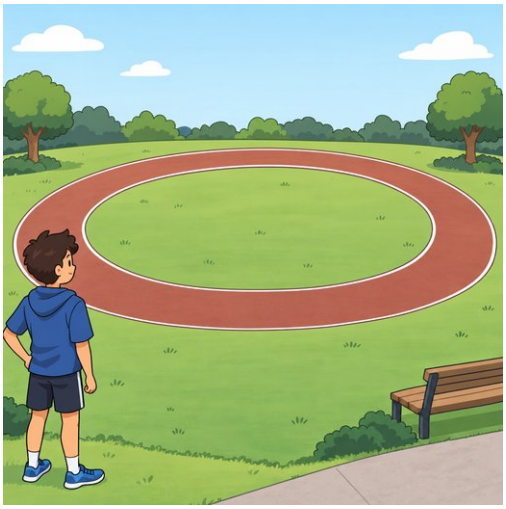
A) 0
B) 8
C) 18
D) 17
E) 1

Problem 9.7 · 5 points — How many integers n make $n^2 + 30n + 81$ the square of an integer?



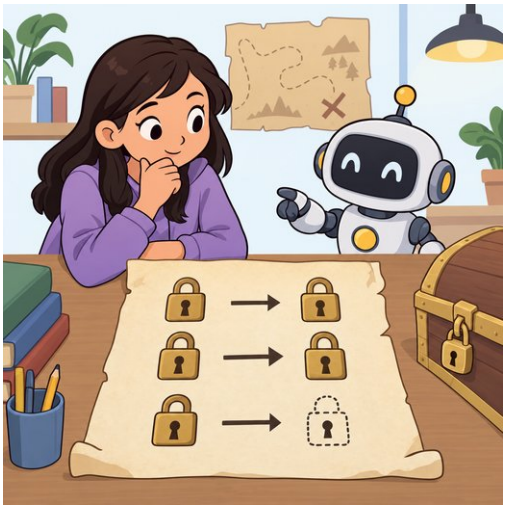
A) 8
B) 10
C) 9
D) 5
E) 3

Problem 9.8 · 4 points — A circular track has two concentric edges. Their circumferences add to 34π metres, and the track's area is 51π square metres. What is the inner edge's circumference?



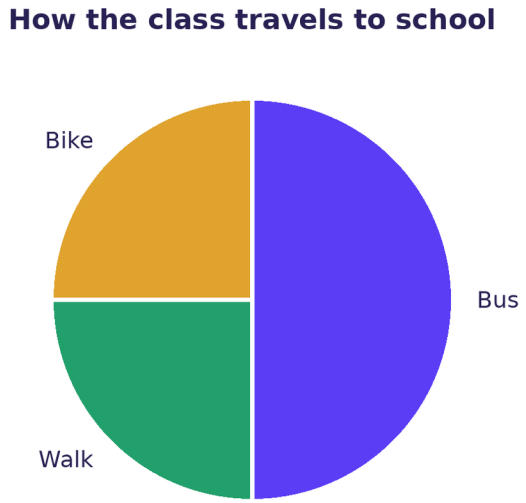
A) 14π metres
B) 20π metres
C) 6π metres
D) 17π metres
E) 28π metres

Problem 9.9 · 5 points — A six-digit lock code has distinct, nonzero digits. Multiplying it by 5 moves its last digit to the front while keeping the other digits in order; what is the sum of the original code’s first and last digits?



A) 8
B) 6
C) 9
D) 12
E) 7

Problem 9.10 · 5 points — Choose any 20 students from this class of 40, then list every possible three-person team with one bus rider, one walker, and one cyclist. Students may appear on several teams. What is the greatest possible number of teams on the list?



A) 250
B) 294
C) 280
D) 288
E) 343

Problem 9.11 · 3 points — Fourteen lily pads sit in a row. A frog paints exactly seven gold, forming exactly three groups of consecutive gold pads, each with at least two pads. How many paintings are possible?

A) 168
B) 120
C) 105
D) 56
E) 360

Problem 9.12 · 4 points — A robot walks on a 3-by-4 grid of intersections, with streets only between horizontal or vertical neighbors. If it must walk every street and return to its start, what is the fewest street segments it can walk?

A) 24
B) 23
C) 20
D) 21
E) 22

Grade 10

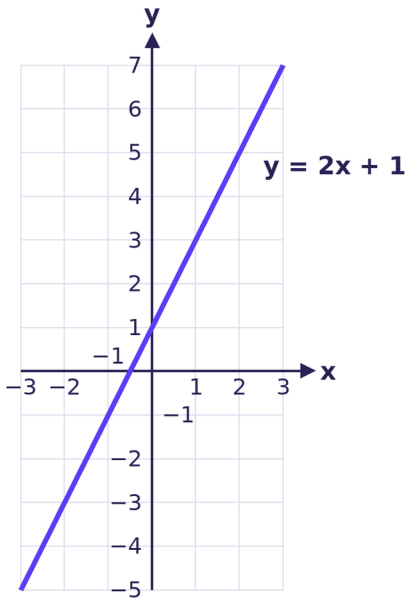
Problem 10.1 · 3 points — What is the minimum value of $x^2 - 6x + 11$?



A) 3
B) 5
C) 2
D) -2
E) 11

Problem 10.2 · 4 points — A square has one side from the drawn line’s y-axis crossing to its grid point with x = 1. Its interior is on the same side of the line as the origin. Which grid point is diagonally opposite the y-axis crossing?

The line y = 2x + 1



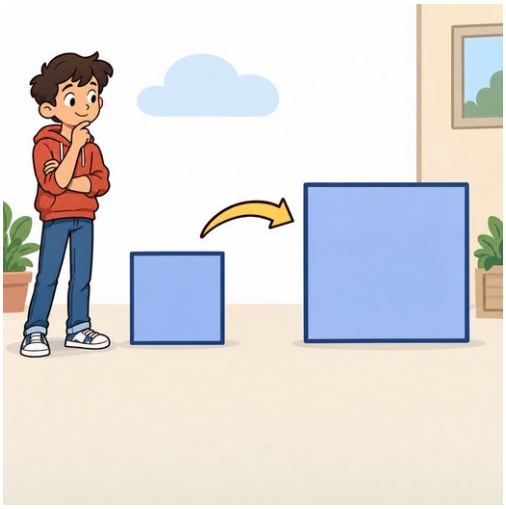
A) (2, 0)
B) (−1, 4)
C) (3, 2)
D) (3, 4)
E) (−2, 2)

Problem 10.3 · 4 points — Four sticker printers make 1, 2, 3, or 4 stickers per minute, and four batteries last 2, 5, 9, or 14 minutes each, regardless of printer. If each printer gets one battery and runs it empty, what is the second-largest total number of stickers possible?



A) 90
B) 91
C) 95
D) 94
E) 92

Problem 10.4 · 3 points — A square is enlarged so that its area’s percent increase is 24 percentage points greater than its perimeter’s percent increase. By what percent did each side increase?



A) 40%
B) 44%
C) 24%
D) 12%
E) 20%

Problem 10.5 · 3 points — A triangle sits inside a larger, similar triangle. The region between them has five-fourths the area of the inner triangle, and their perimeters add to 130 cm. What is the larger perimeter?



A) 65 cm
B) 650/9 cm
C) 90 cm
D) 78 cm
E) 52 cm

Problem 10.6 · 4 points — Let A = sin 15° + cos 75° and B = sin 75° + cos 15°. What is A/B + B/A?



A) 4/√3
B) 2
C) 2 + √3
D) 4
E) 2 − √3

Problem 10.7 · 4 points — A convex polygon is cut into triangles by noncrossing diagonals. Four triangles have no side on the polygon’s boundary, and five have exactly one boundary side. How many sides does the polygon have?

A) 17
B) 15
C) 14
D) 16
E) 18

Problem 10.8 · 5 points — Let $N = 2^5 \cdot 3^5 \cdot 5^5 \cdot 7^5$. How many positive divisors d of N have the property that both d and N/d have a perfect-square number of positive divisors?



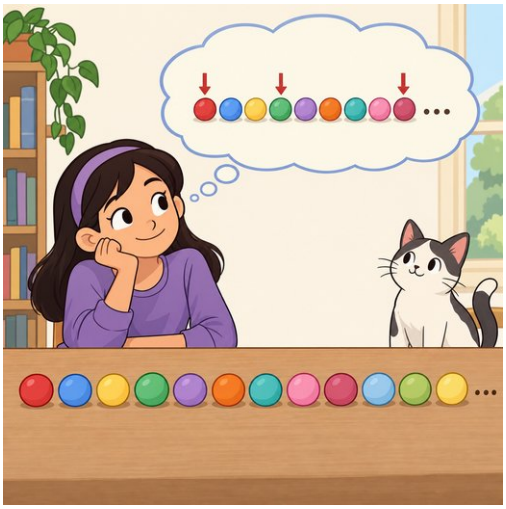
A) 126
B) 96
C) 120
D) 90
E) 36

Problem 10.9 · 5 points — What is the remainder when $1^1 + 2^2 + 3^3 + \cdots + 2020^{2020}$ is divided by 7?



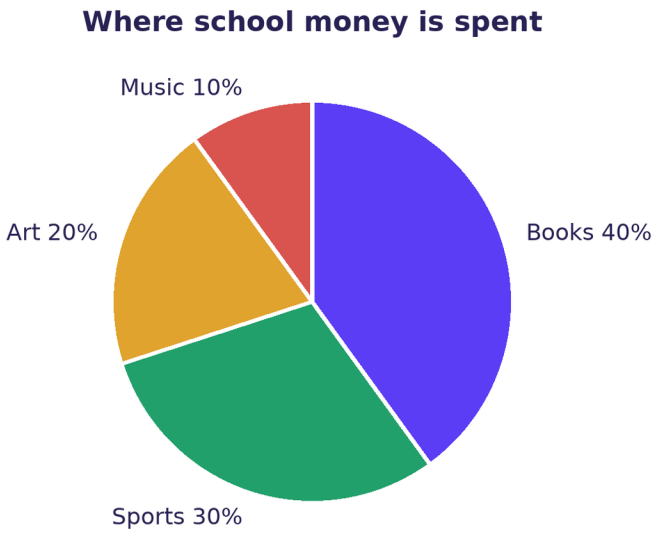
A) 5
B) 2
C) 0
D) 6
E) 1

Problem 10.10 · 5 points — Nora’s sequence of positive integers has this property: in any four consecutive terms, the product of the outer two equals the product of the inner two. If the 1st, 4th, and 9th terms are 2, 36, and 2592, what is the first position whose term is both a perfect cube and divisible by 2^100 × 3^101?



A) 204
B) 206
C) 203
D) 201
E) 202

Problem 10.11 · 5 points — Three points are picked independently and uniformly from inside the pie chart. Given that they land in three different slices, what is the probability that one lands in books?



A) 75%
B) 88%
C) 78.4%
D) 90%
E) 40%

Problem 10.12 · 3 points — Five cards start in the row 1, 2, 3, 4, 5. On each of exactly two turns, you choose any three positions and rotate their cards in either direction; which final row is impossible?



A) 2, 3, 4, 1, 5
B) 3, 1, 2, 4, 5
C) 2, 1, 4, 3, 5
D) 2, 3, 4, 5, 1
E) 1, 2, 3, 4, 5

Grade 11

Problem 11.1 · 3 points — An increasing exponential function f has f(0) + f(2) = 25 and f(1) = 10. What is f(3)?



A) 5/2
B) 20
C) 40
D) 30
E) 35

Problem 11.2 · 3 points — For $x > 0$ with $x \neq 1$, suppose $\log_x 16 = \log_2 x$. What is the sum of all possible values of x?



A) 0
B) 1/4
C) 4
D) 1
E) 17/4

Problem 11.3 · 3 points — Twenty dots lie on the graph of a quadratic function, with equally spaced x-coordinates from left to right. The first three are (5, 41), (8, 110), and (11, 215); what is the height of the 20th dot?

A) 7502
B) 1352
C) 8261
D) 7508
E) 6791

Problem 11.4 · 4 points — For which real value of t does $(x^2 - 2tx + 1)(x^2 - 2x + t^2) = 0$ have exactly one distinct real solution for x?



A) $t = -1$ or $t = 1$

B) $t = -1$

C) $t = 1$

D) No real value of t

E) $-1 < t < 1$

Problem 11.5 · 5 points — Real numbers a, b, and c satisfy $a+b+c=0$ and $a^2+b^2+c^2=6$. What is the greatest possible value of $a^6+b^6+c^6$?



A) 64

B) 66

C) 72

D) 78

E) 54

Problem 11.6 · 4 points — Cups numbered 1, 2, 3, ... hold 2, 1, 1/2, 1/4, ... in order. Ada marks every even-numbered cup, and Ben marks every cup numbered by a multiple of 3; what is the sum in cups marked by exactly one of them?



A) 4/7
B) 116/63
C) 40/21
D) 16/9
E) 4/3

Problem 11.7 · 4 points — A bag holds 5 red and 5 blue marbles. You draw without replacement until all the marbles of one color have been drawn. What is the expected number of marbles left in the bag?



A) 5/3
B) 2
C) 5/6
D) 3/2
E) 1

Problem 11.8 · 5 points — How many circular arrangements of the letters AABBCCDD have no identical letters next to each other? Count rotations as the same, but do not identify mirror images.



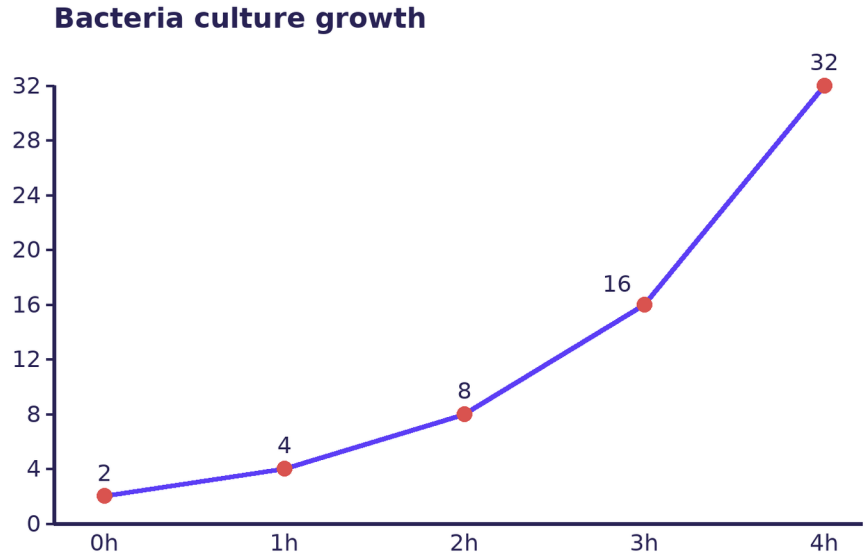
A) 315
B) 96
C) 318
D) 93
E) 744

Problem 11.9 · 5 points — A lock shows codes 3, 10, 23, 42, 67 for keys 1, 2, 3, 4, 5. Its hidden program is a polynomial with integer coefficients, but its degree is unknown; which code could key 6 show?



A) 218
B) 122
C) 138
D) 128
E) 158

Problem 11.10 · 5 points — If the chart’s exponential trend continues, what is the first whole-number hour after hour 4 when the bacteria count is a perfect fourth power and its increase since hour 4 is divisible by 49?



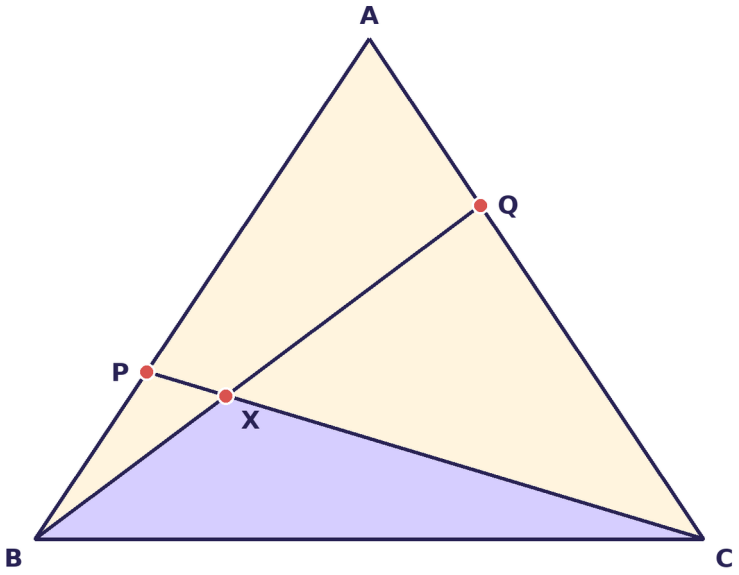
A) 25
B) 67
C) 7
D) 46
E) 68

Problem 11.11 · 3 points — Nine lamps, numbered 1 through 9 clockwise around a circle, start off. A move flips any three consecutive lamps; which set could be exactly the lamps left on?

A) {3, 4, 6, 7}
B) {1, 3, 6, 9}
C) {1, 2, 6, 8}
D) {1, 2, 3, 4}
E) {2, 4, 6, 7}

Problem 11.12 · 4 points — On a triangular mural ABC, P lies on AB with AP:PB = 2:1, and Q lies on AC with AQ:QC = 1:2. If CP and BQ cross at X, what fraction of the mural is triangle BXC?

The mural ABC



A) 1/3
B) 4/7
C) 5/7
D) 1/7
E) 2/7

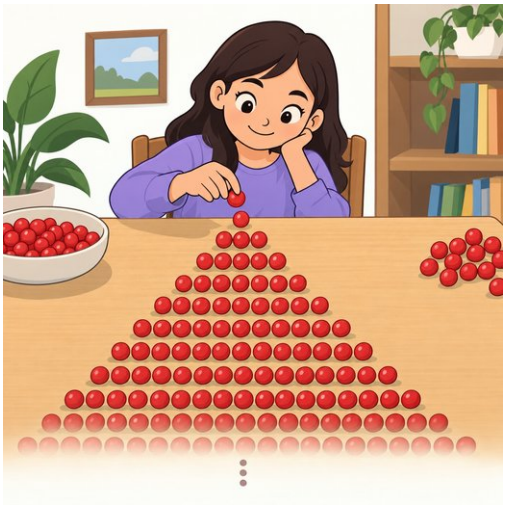
Grade 12

Problem 12.1 · 3 points — A complex number z has $|z| = 2$, and $(z + 1)^2$ and $(z - 1)^2$ have the same real part. What is $(z + i)^2 + (z - i)^2$?



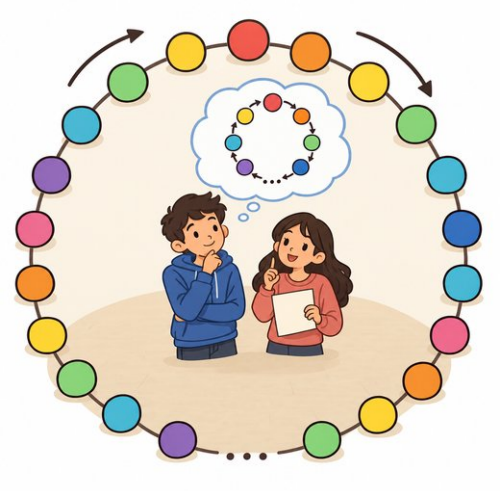
A) -6
B) 10
C) -10
D) -8
E) 6

Problem 12.2 · 5 points — Split the first N positive odd integers into two nonempty consecutive blocks. If the second block sums to exactly one more than the first, what is the smallest possible N greater than 100?



A) 408
B) 577
C) 578
D) 239
E) 99

Problem 12.3 · 3 points — Let g move each number in {0, 1, …, 2025} one step forward around a circle, wrapping from 2025 to 0. How many permutations f have no fixed points and satisfy $f(g(x)) = g^{-1}(f(x))$ for every x?



A) 0
B) 1012
C) 2026
D) 2
E) 1013

Problem 12.4 · 4 points — For which positive value of a does $|x^2 - 4x + 3| = ax$ have exactly three distinct real solutions?



A) 1/2
B) $4 + 2\sqrt{3}$
C) $4 - 2\sqrt{3}$
D) $2\sqrt{3} - 3$
E) 1

Problem 12.5 · 4 points — Let $x > 1$ satisfy $x + 1/x = 3$, and let log mean base 10. Which single logarithm equals $\log(x^3 - 1) - \log(x - 1) + \log(1 + x^{-3}) - \log(1 + x^{-1})$?



A) log 3
B) log 9
C) log 7
D) log 8
E) log(1/8)

Problem 12.6 · 5 points — Let a, b, and c be the roots of $x^3 - 3x + 1 = 0$. What is $a^8/(3a^2 - 3) + b^8/(3b^2 - 3) + c^8/(3c^2 - 3)$?



A) 0
B) 28
C) 6
D) 186
E) -27

Problem 12.7 · 3 points — Real numbers a, b, and c satisfy $a+b+c=4$ and $a^2+b^2+c^2=14$. If $u=(a-b,\ b-c,\ c-a)$ and $v=(b-c,\ c-a,\ a-b)$, what is $u\cdot v$?



A) -21
B) 0
C) 13
D) -13
E) -26

Problem 12.8 · 4 points — A robot travels within the closed first quadrant from (2,7) to (8,3). It must visit the y-axis and then the x-axis; what is the shortest possible distance it can travel?



A) √53 + √73
B) 2√13
C) 2√29
D) 2√34
E) 10√2

Problem 12.9 · 4 points — A necklace has 4 black beads and 8 white beads, with no two black beads touching. How many arrangements are possible if two arrangements count as the same exactly when one is a rotation of the other?

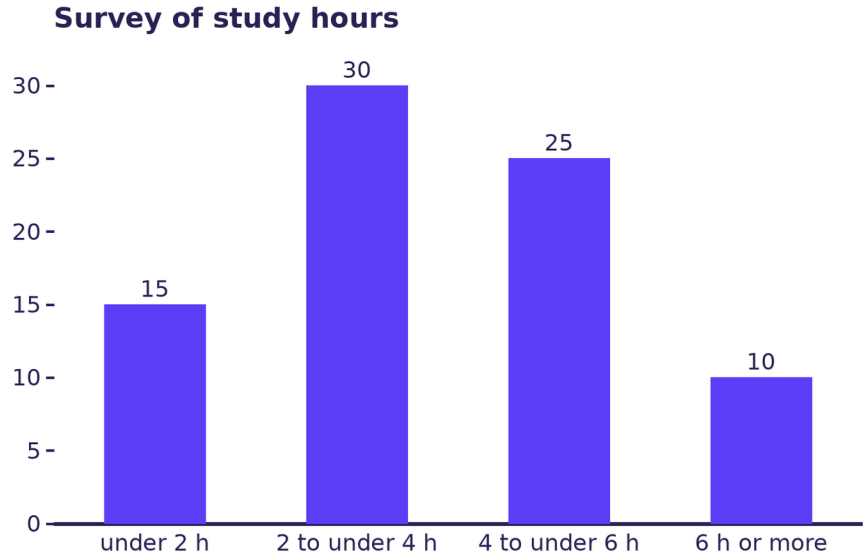
A) 10
B) 8
C) 35
D) 105
E) 9

Problem 12.10 · 5 points — When an integer’s fourth power is divided by 65, which of these numbers cannot be the remainder?



A) 56
B) 61
C) 26
D) 35
E) 16

Problem 12.11 · 5 points — Four students are chosen at random without replacement. If their study hours fall in exactly three bars of the chart, what is the probability that the bar represented twice is 4 to under 6 hours or 6 hours or more?



A) 1/2
B) 303/724
C) 7/16
D) 8/19
E) 23/59

Problem 12.12 · 3 points — A triangular LED display starts with only its top LED lit (row 0). Each new row starts dark, and every lit LED in the preceding row toggles its two children; how many LEDs are lit in row 2026?



A) 256
B) 1014
C) 128
D) 512
E) 2027

Answer key

1.1: C
1.2: E
1.3: A
1.4: D
1.5: C
1.6: B
1.7: A
1.8: E
1.9: D
1.10: B
1.11: A
1.12: B
2.1: C
2.2: B
2.3: E
2.4: A
2.5: D
2.6: C
2.7: B
2.8: A
2.9: D
2.10: A
2.11: B
2.12: E
3.1: C
3.2: E
3.3: D
3.4: C
3.5: E
3.6: D
3.7: A
3.8: B
3.9: B
3.10: A
3.11: A
3.12: B
4.1: C
4.2: C
4.3: B
4.4: E
4.5: A
4.6: A
4.7: E
4.8: D
4.9: D

4.10: B
4.11: A
4.12: B
5.1: C
5.2: E
5.3: A
5.4: D
5.5: C
5.6: D
5.7: B
5.8: A
5.9: E
5.10: B
5.11: A
5.12: B
6.1: C
6.2: E
6.3: D
6.4: B
6.5: C
6.6: D
6.7: A
6.8: A
6.9: B
6.10: E
6.11: A
6.12: B
7.1: C
7.2: E
7.3: C
7.4: B
7.5: A
7.6: D
7.7: D
7.8: A
7.9: B
7.10: A
7.11: B
7.12: E
8.1: C
8.2: C
8.3: D
8.4: E
8.5: B
8.6: D
8.7: B

8.8: A
8.9: A
8.10: B
8.11: A
8.12: E
9.1: C
9.2: D
9.3: C
9.4: B
9.5: E
9.6: D
9.7: B
9.8: A
9.9: A
9.10: B
9.11: A
9.12: E
10.1: C
10.2: C
10.3: E
10.4: E
10.5: D
10.6: D
10.7: A
10.8: B
10.9: B
10.10: A
10.11: B
10.12: A
11.1: C
11.2: E
11.3: D
11.4: C
11.5: B
11.6: D
11.7: A
11.8: B
11.9: A
11.10: B
11.11: A
11.12: E
12.1: C
12.2: B
12.3: E
12.4: C
12.5: D

12.6: B
12.7: D
12.8: E
12.9: A
12.10: A
12.11: B
12.12: A